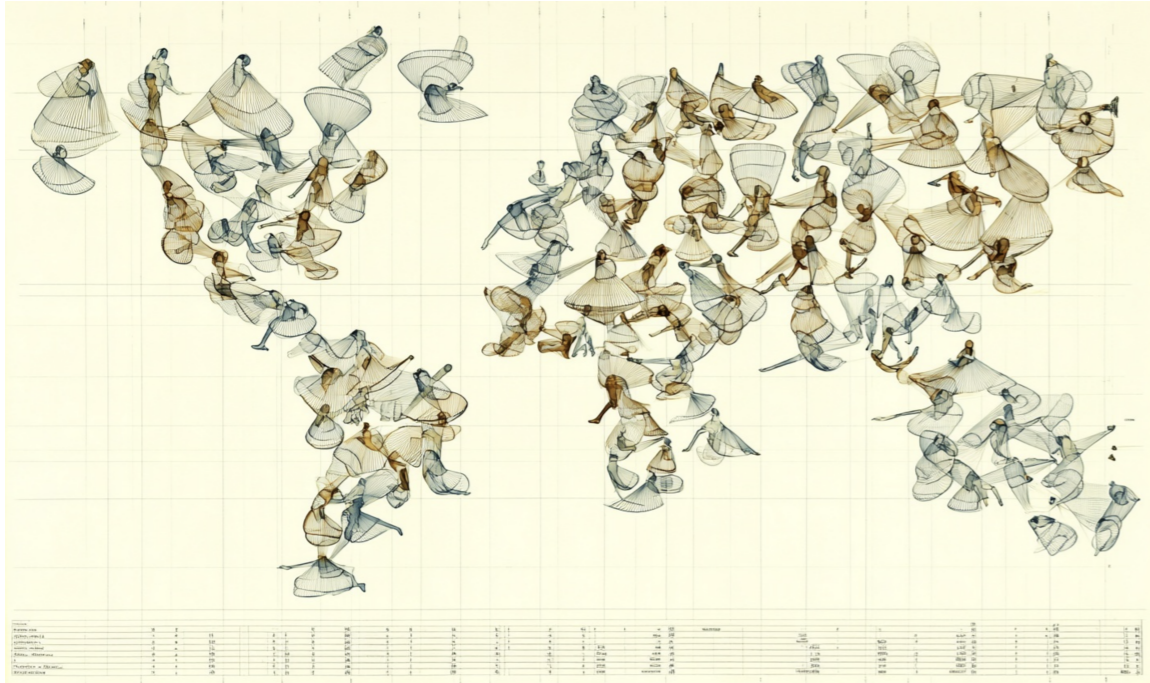


An Atlas of Risk Measures

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Abstract



An Atlas of Risk Measures organizes risk measures along two important dimensions: how a measure treats diversification, and how well it behaves under limits. The Atlas unifies expected value, spectral measures, coherent and convex risk measures, Value at Risk, and stranger finitely additive objects that usually stay offstage. The Atlas turns a technically dense literature into a map: it shows where the main examples live, what assumptions move you from one region to another, and why convex duality, the Fatou property, and law invariance matter so much. The result is an appealing visual tour of modern risk measure theory from an actuarial, pricing-centered point of view.

1. The Atlas and Associated Figures

1.1 Atlas of Risk Measures

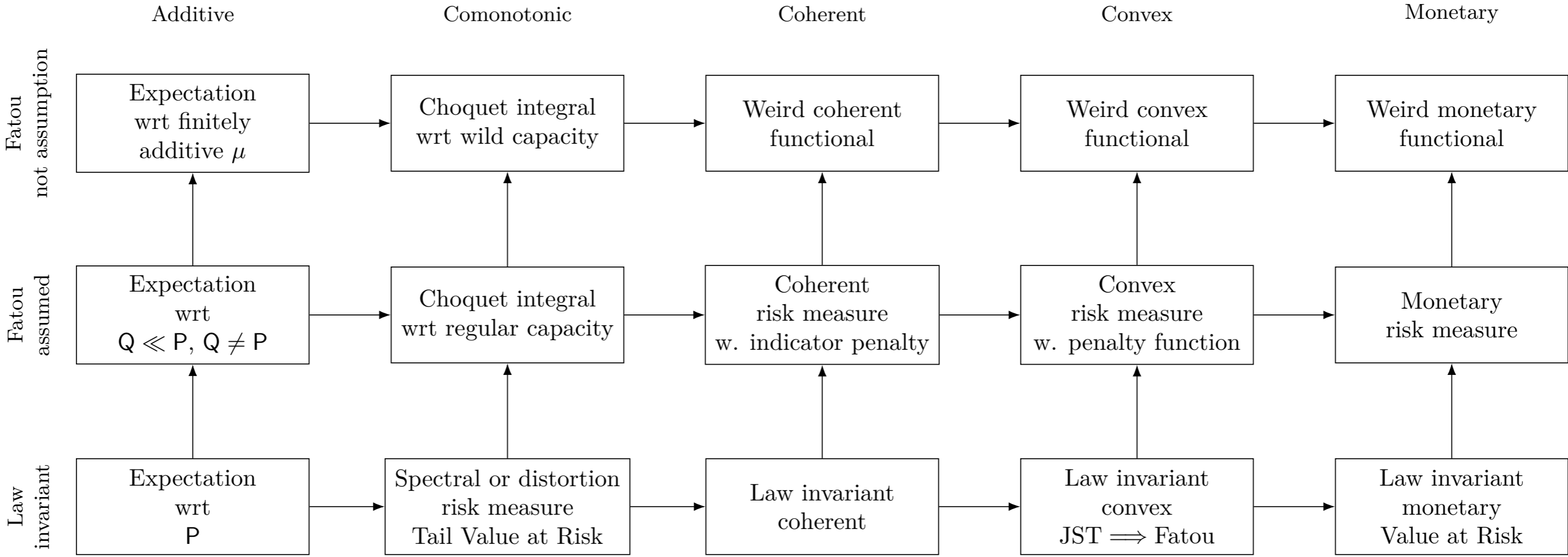


Figure 1: Atlas of risk measures. Continuity increases down each column and diversification additivity becomes more flexible across each row.

1.2 Populated Atlas

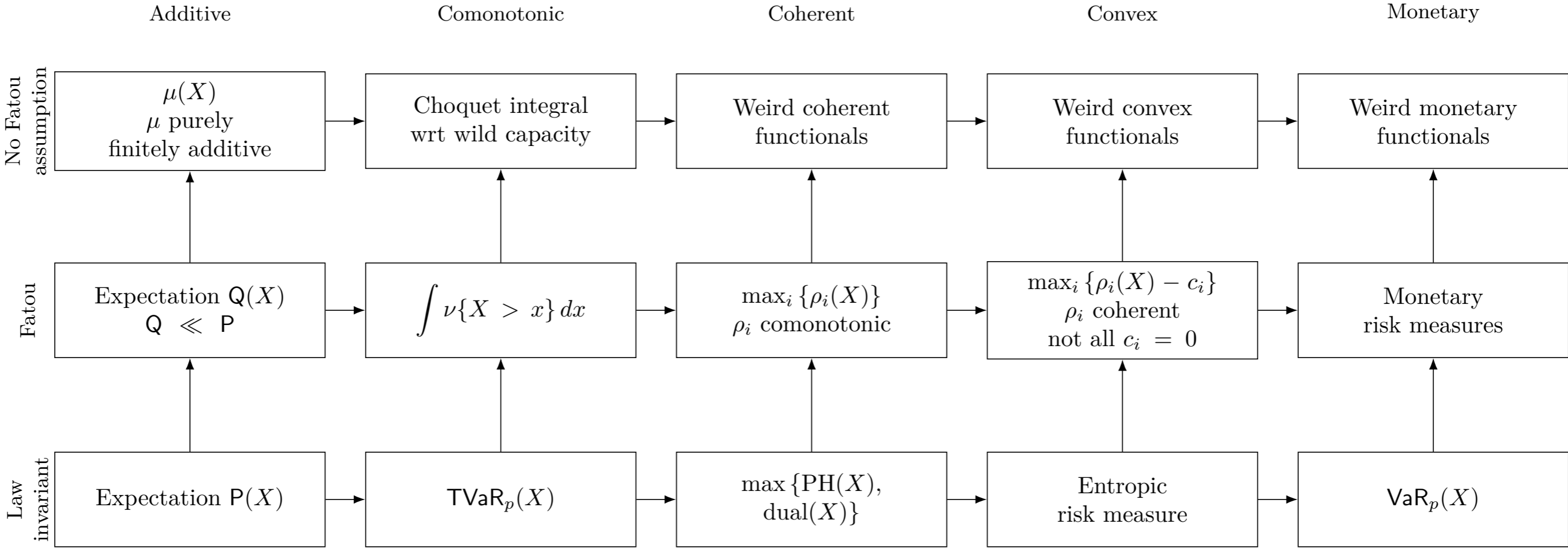


Figure 2: Populated atlas.

1.3 Relationships Between Properties of Risk Measures

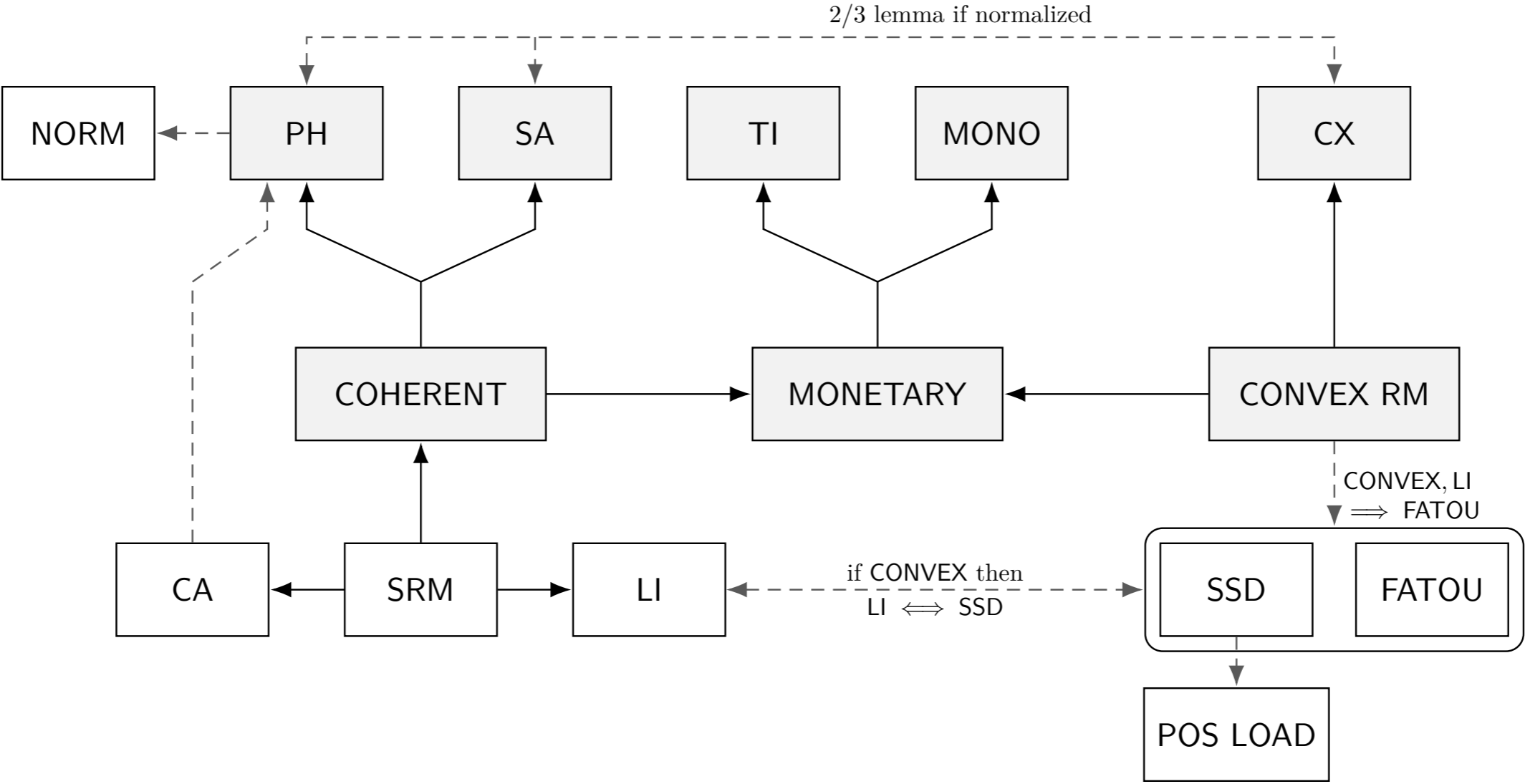


Figure 3: Relationships between properties of risk measures. Source Mildenhall and Major (2022).

1.4 Geometric Representations of Penalty Function Epigraphs

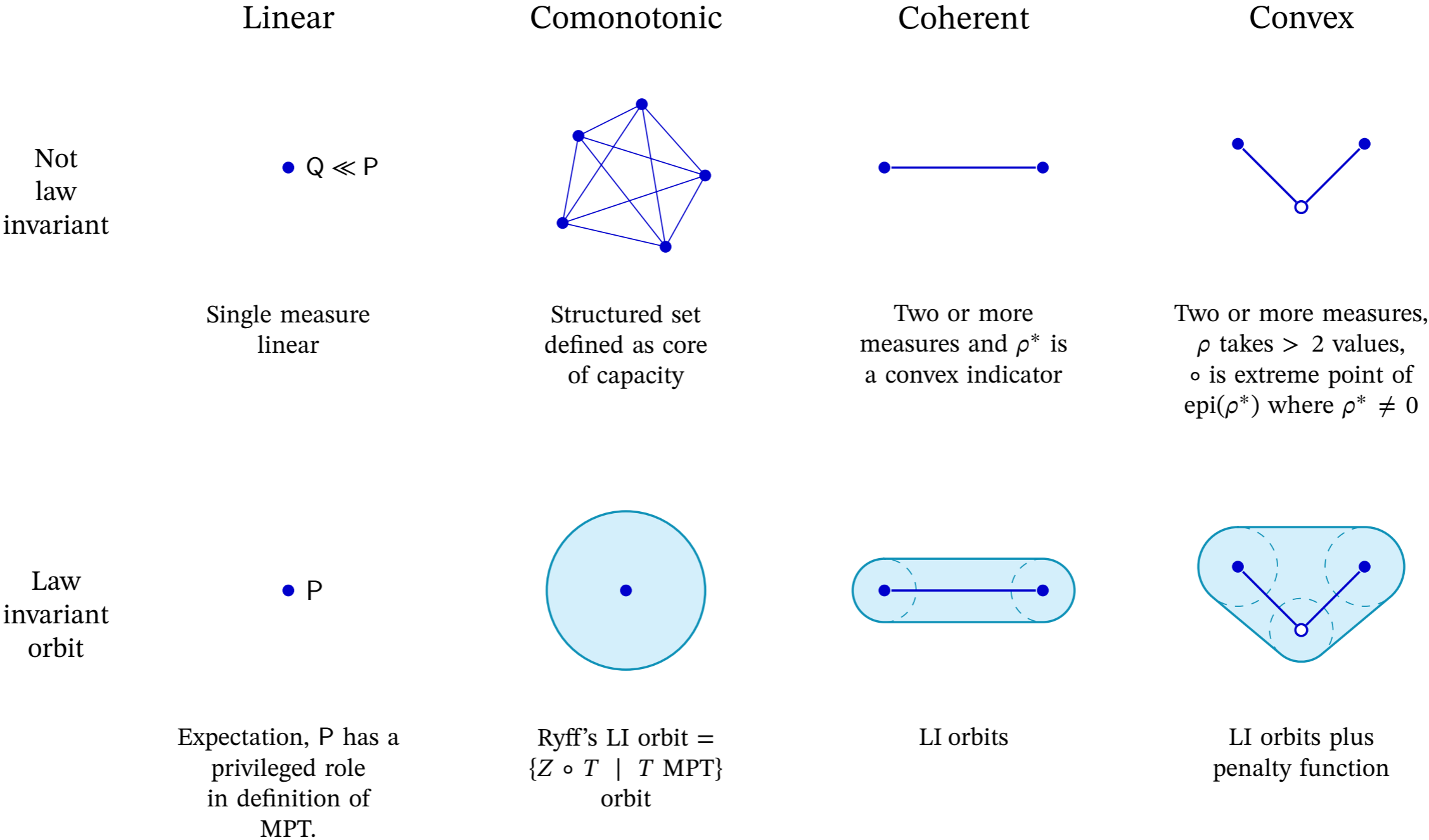


Figure 4: Geometric representations of penalty function epigraphs, $\text{epi}(\rho^*)$, across risk measure classes. The top row illustrates general dual domains, and the bottom row the symmetrically expanded law invariant domain. The columns show increasingly general algebraic additivity conditions, from linearity on the left to convexity on the right.

1.5 Atlas Topology

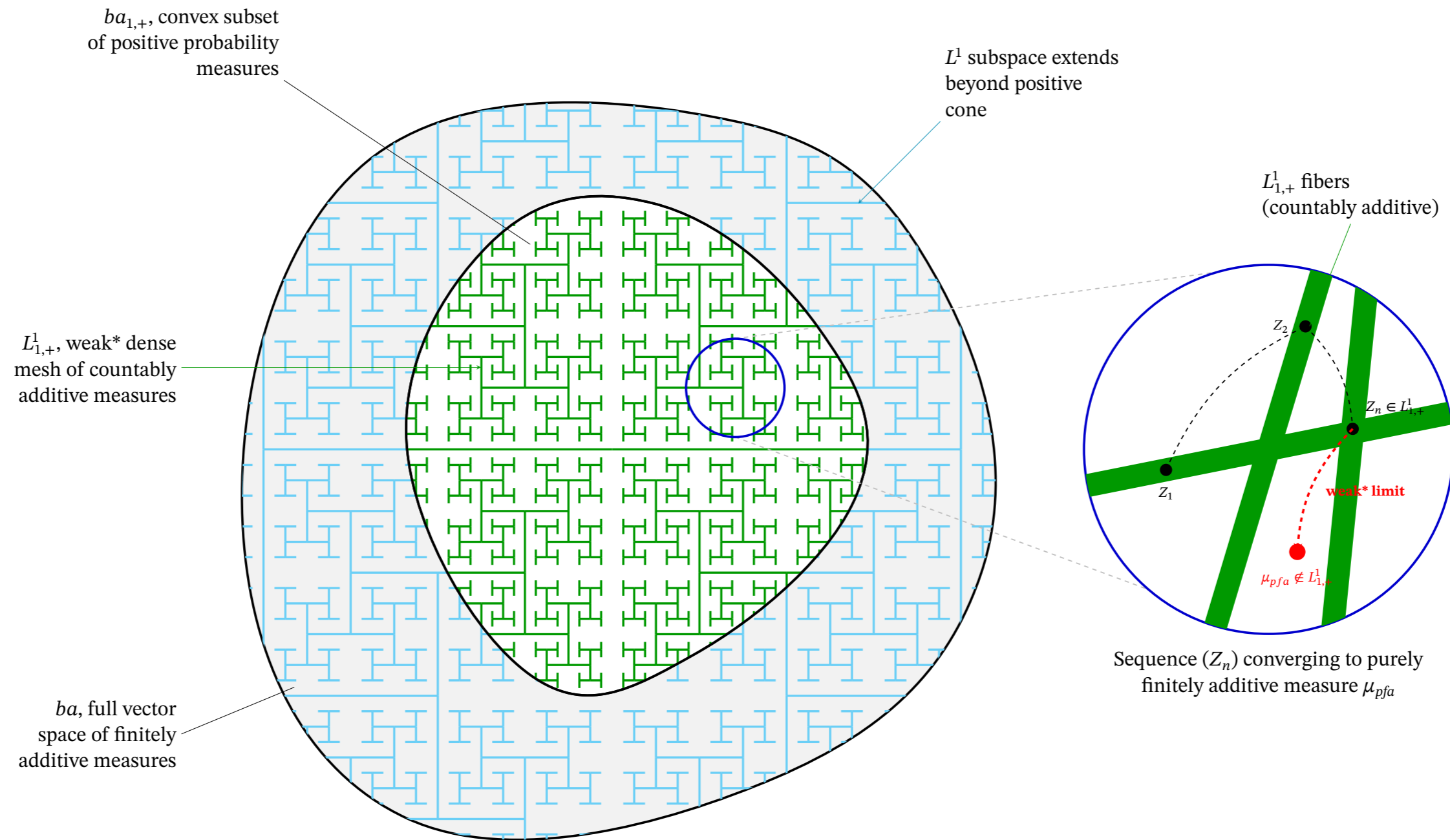


Figure 5: Topology of the dual space $ba = (L^\infty)^*$. The green sub-region shows the probability measures (positive, total measure 1) within ba . The fractal H -mesh cartoon illustrates the weak* dense subset of countably additive measures (L^1) within the broader space of finitely additive measures (ba). The magnified view demonstrates a sequence of probability densities ($Z_n \in L^1_{1,+}$) converging in the weak* topology to a purely finitely additive measure (μ_{pfa}), illustrating the necessity of the extended space $ba_{1,+}$ for representing general risk measures.

1.6 Construction

For a convex, lsc, monetary risk measure ρ on L^∞ , with α convex and lsc.

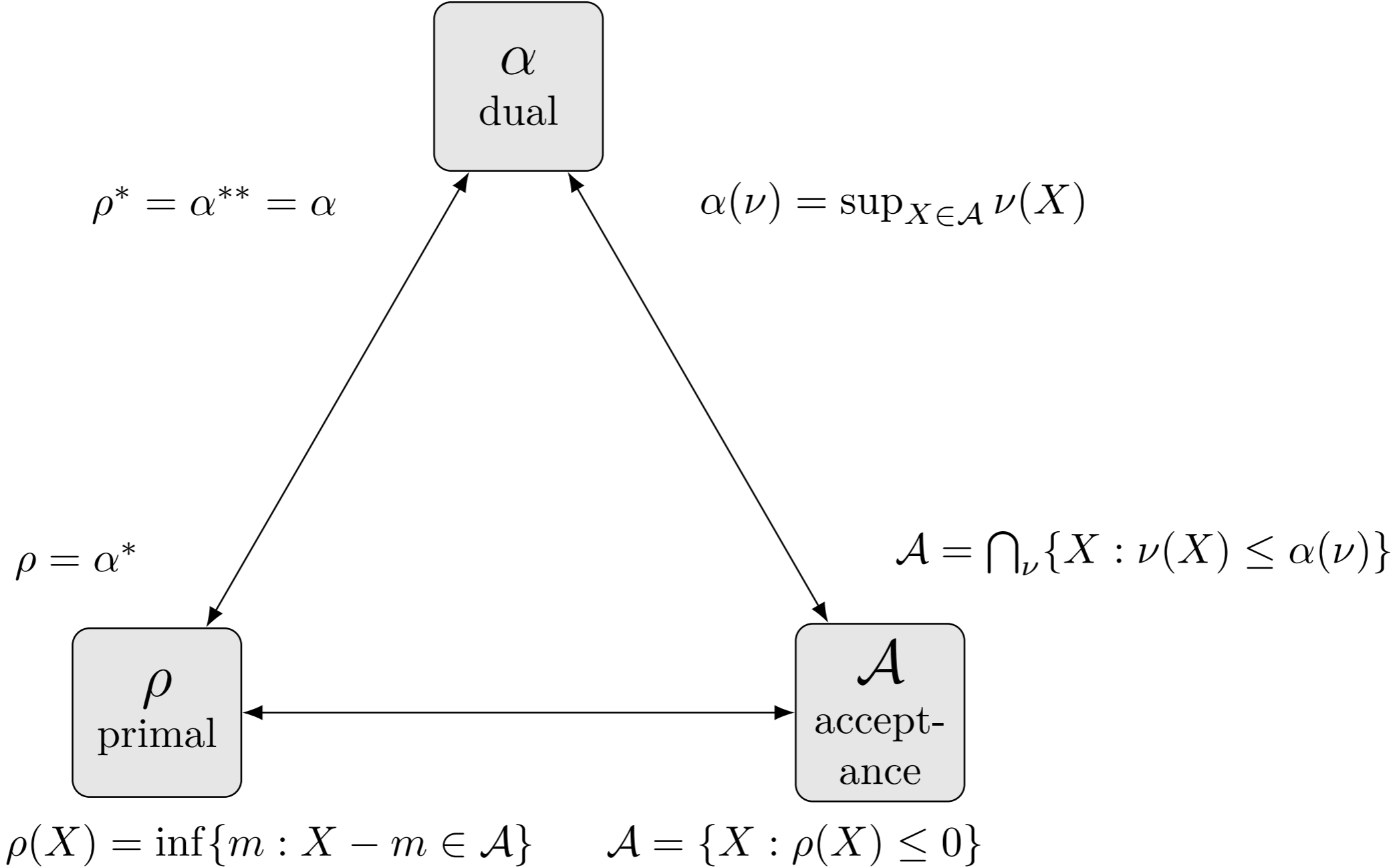


Figure 6: Starting from a convex, lsc α .

1.7 Construction (Arbitrary α)

Starting from arbitrary α convex and lsc, creating α_{\min} .

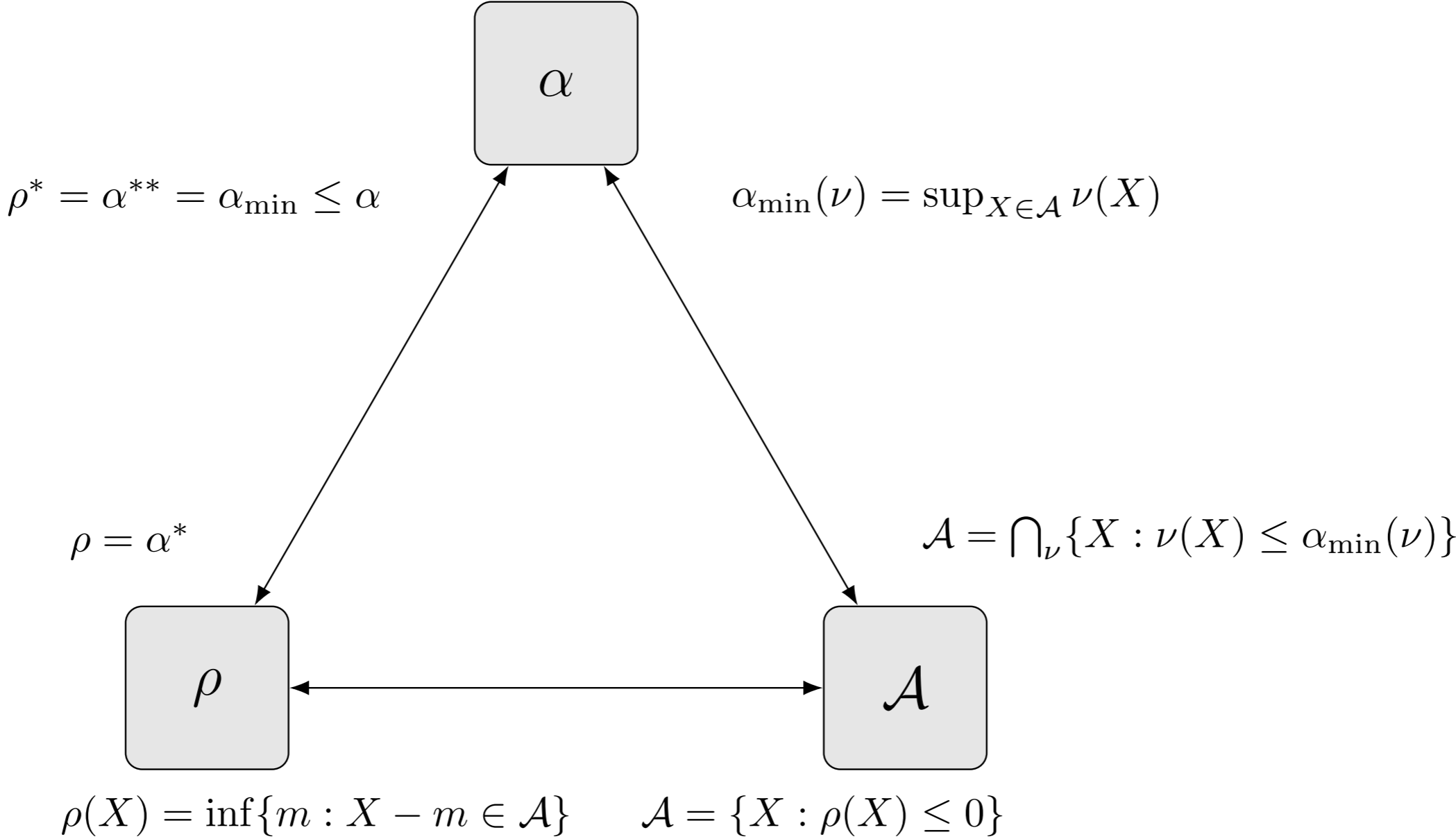


Figure 7: Starting from an arbitrary α .

1.8 Primal axioms and dual restrictions

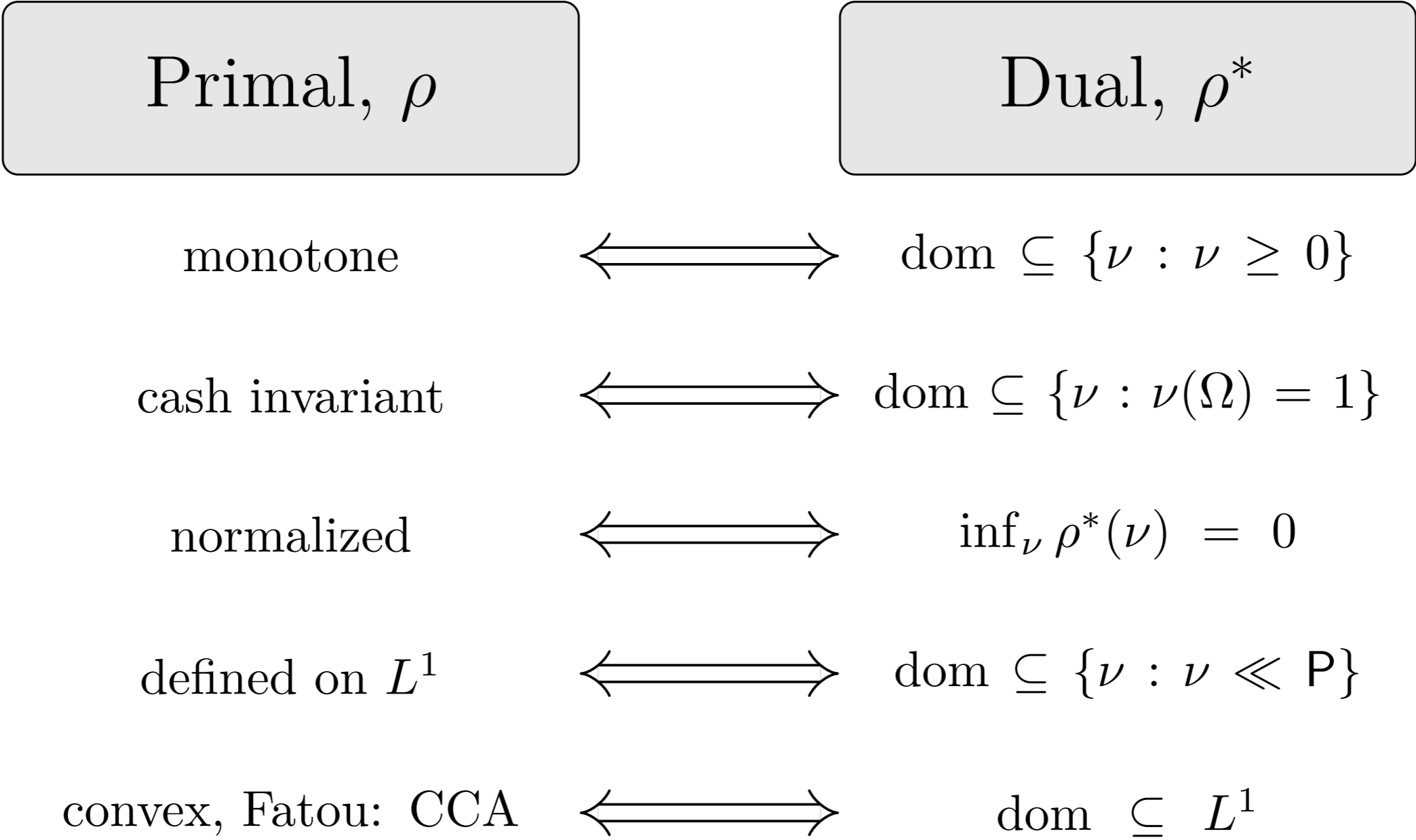


Figure 8: Primal axioms and corresponding restrictions on the dual function.

1.9 Acceptance Sets

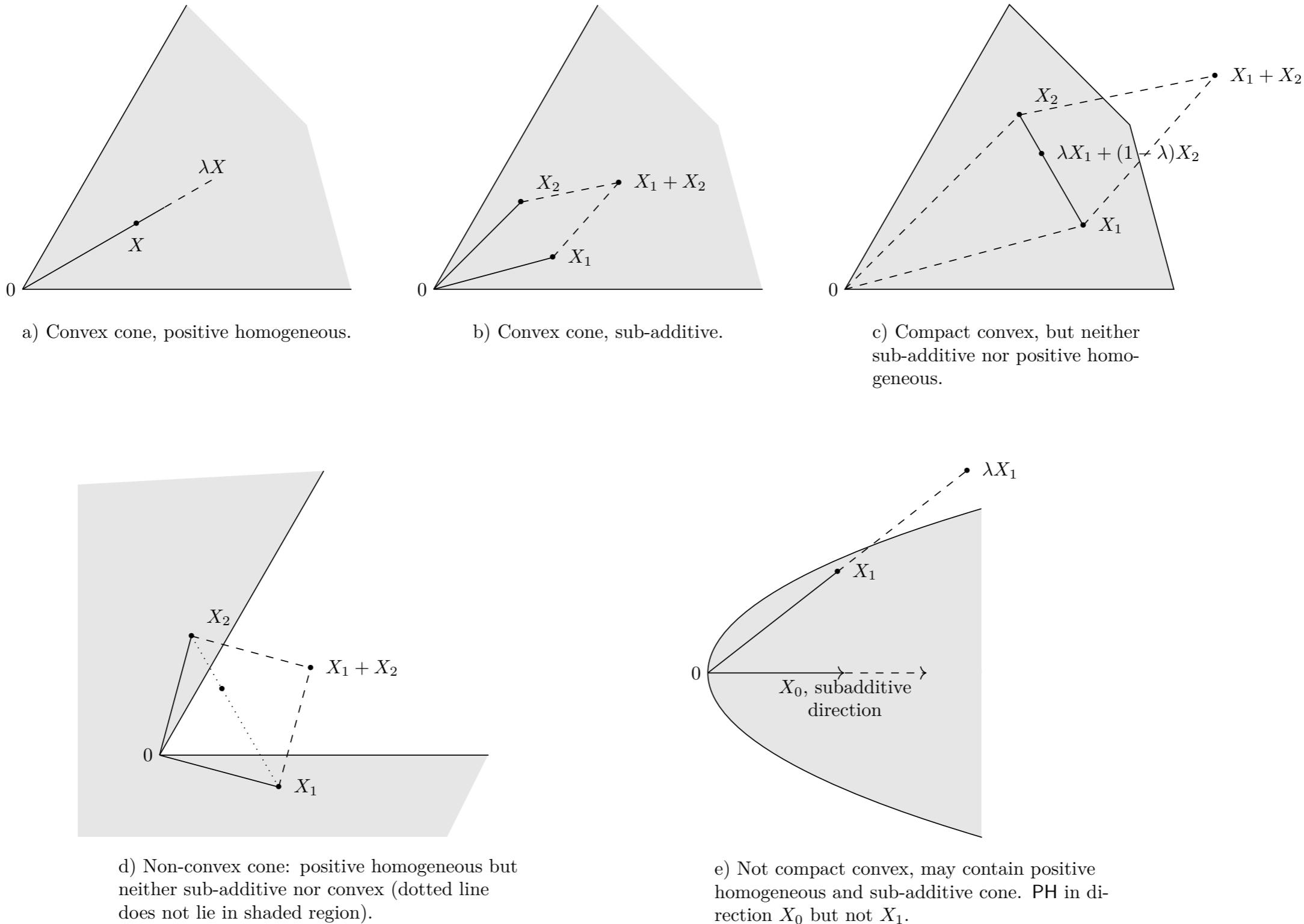


Figure 9: Acceptance sets and corresponding risk measure properties. The shaded region represents the acceptance set \mathcal{A} . Black lines represent the boundary; unbounded region have no boundary line. Only (c) is bounded. The origin is marked with 0. (a) A convex cone is positive homogenous, the line λX is contained in the shaded region \mathcal{A} and (b) is also sub-additive, the sum of two points in \mathcal{A} is again in \mathcal{A} . (c) A compact convex \mathcal{A} is neither positive homogeneous nor sub-additive, but the line joining two points in \mathcal{A} is in \mathcal{A} . (d) A non-convex cone is positive homogeneous but not sub-additive. (e) A non-compact convex \mathcal{A} can be

1.10 Continuity Hierarchy

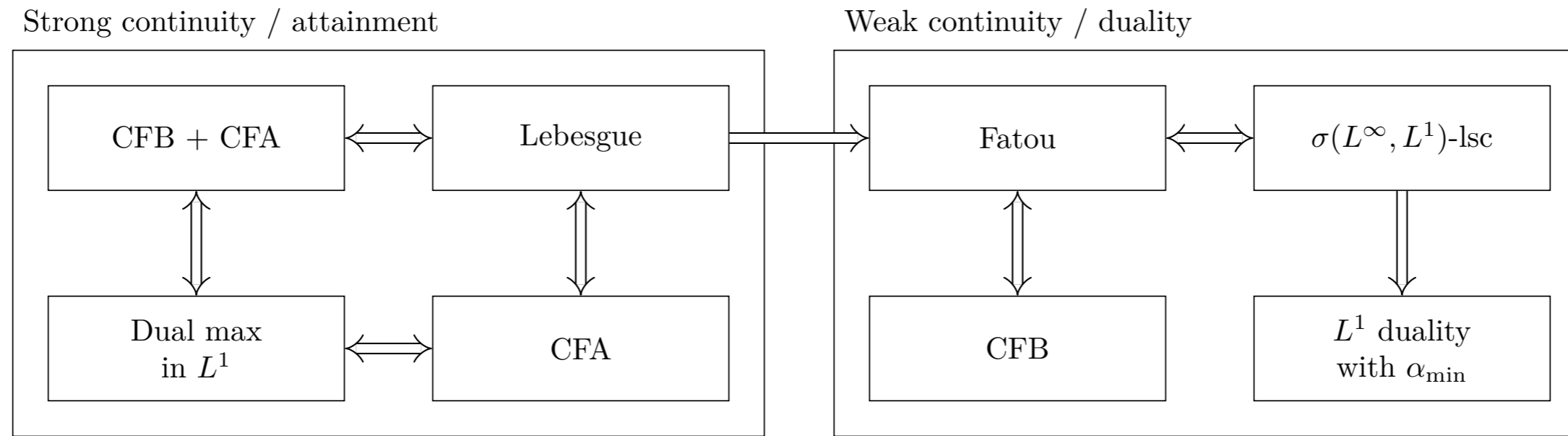


Figure 10: Relationship between continuity concepts for a convex monetary risk measure.

1.11 Convergence Concepts

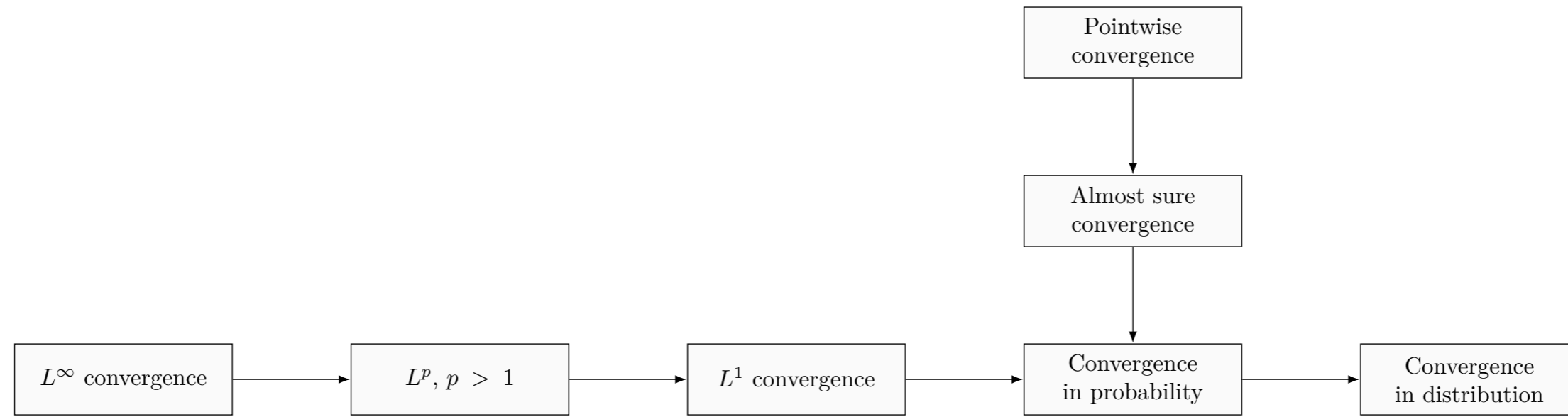


Figure 11: Convergence concepts for random variables.

2. How To Do Other Things

2.1 Bumpf

Too many actuarial pricing models

- over-weight tail catastrophe risk, and
- under-weight earnings-hit volatility risk.

This behavior drives portfolio steering and reinsurance purchase decisions materially at odds with management's risk preferences.

2.2 Diagnosis

Management's complaint is entirely explained by an assumption that the cost of capital is constant.

The assumption is often implicit and hidden.

2.3 Prescription

Stop assuming the cost of capital is constant!

There are other good alternatives ... enter Spectral Risk Measures (SRMs).

2.4 Example: Pricing

2.4.1 Keep these numbers in mind!

- Market pricing for the total portfolio earns a 15% return on a fully capitalized basis
- Premium P , capital Q , total assets $a = P + Q = \max(L) = 100$, and return $r = 0.15$ are related by

$$P = E[L] + r(a - P) \implies P = vE[L] + d \max(L)$$

where $v = 1/(1 + r)$ and $d = r/(1 + r)$ are the discount rate and factor

- Facts imply $P = 53.565$, loss ratio 87%, and premium to surplus ratio of 1.15:1

3. Another Section

3.1 Slide

Left text column

1. A
2. B
3. C

Right text

- one
- two
- three

3.1.1 After blue bit

- After bullet
- Mildenhall (2022), Mildenhall and Major (2022)

4. References

4.1 References

Mildenhall, Stephen J., 2022, Similar Risks Have Similar Prices: A Useful and Exact Quantification, *Insurance: Mathematics and Economics* 105, 203–210.

Mildenhall, Stephen J., and John A. Major, 2022, *Pricing Insurance Risk: Theory and Practice* (John Wiley & Sons, Inc.).